



## Diffusion Imaging

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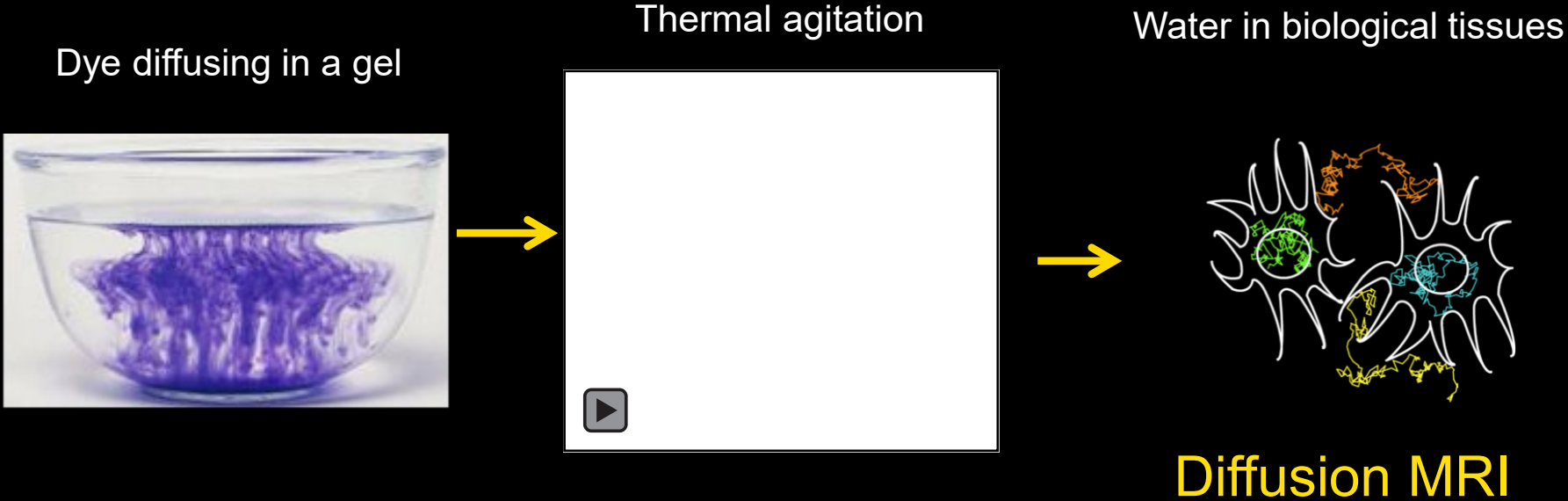
Microstructure Imaging Lab

Department of Radiology, Univ of Iowa

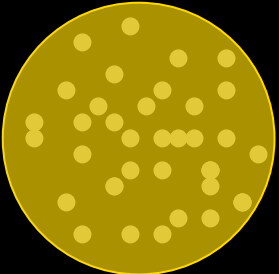
- What is diffusion and why do we care ?
- How to model diffusion signal?
- What are the pitfalls of modeling ?
- Fiber Tracking using diffusion MRI
- How to pre-process diffusion data?
- How to extract biologically meaningful diffusion measures ?

# What is diffusion and why do we care ?

➤ “Diffusion”



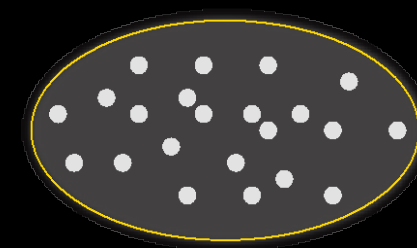
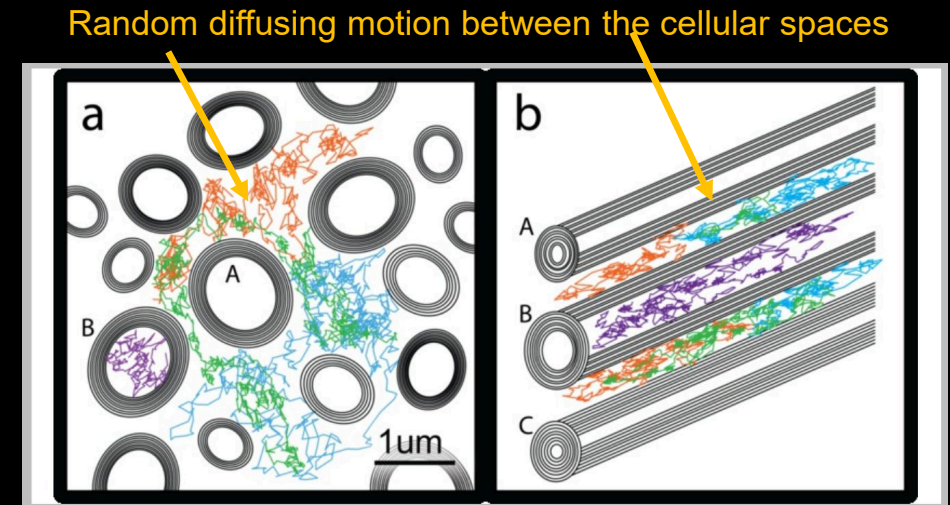
➤ In Unrestricted Medium:



➤ Isotropic Diffusion

# What is diffusion and why do we care ?

- In Biological Tissues:
  - Presence of tissue boundaries
  - Random walk get modified
  
- Restricted Medium :
  - Isotropic Diffusion not possible

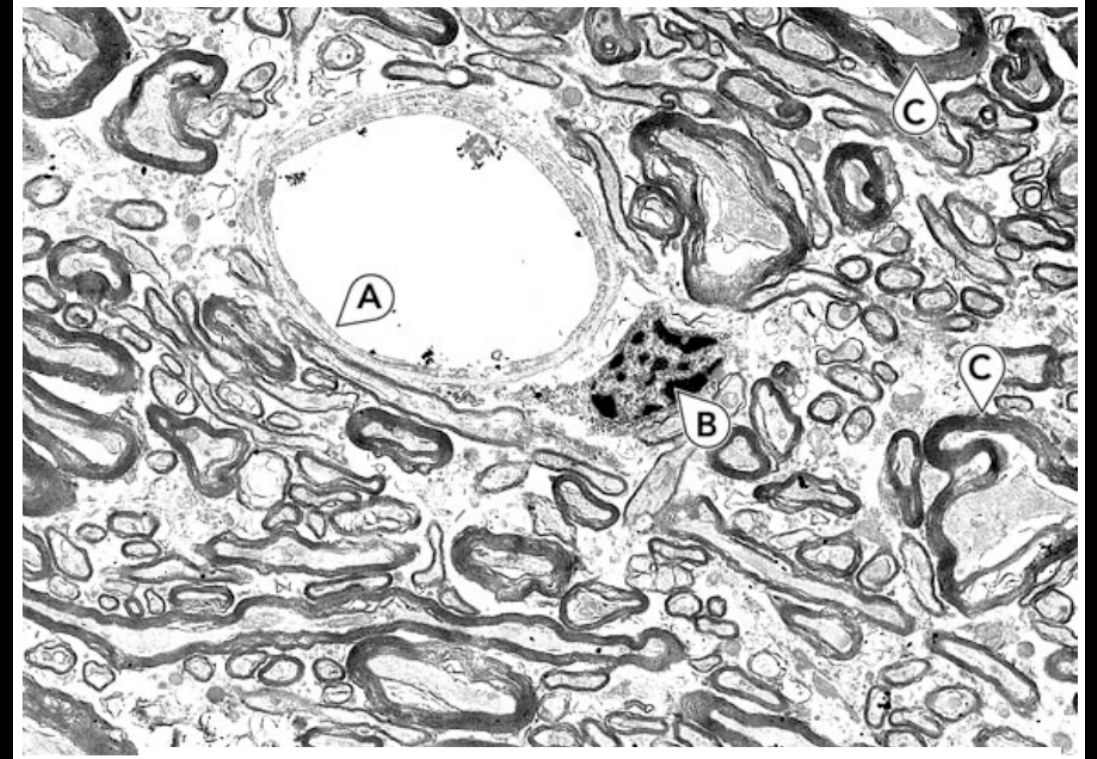


Anisotropic Diffusion

- Patterns of water diffusion in tissue reflect the microstructure of the medium

# What is diffusion and why do we care ?

- In Biological Tissues:
  - Membrane permeability
  - Macromolecules
  - Packing density
  - Compartment size



Sensitizing the MRI signal to water diffusion is a way to indirectly get information about tissue microstructure and its changes

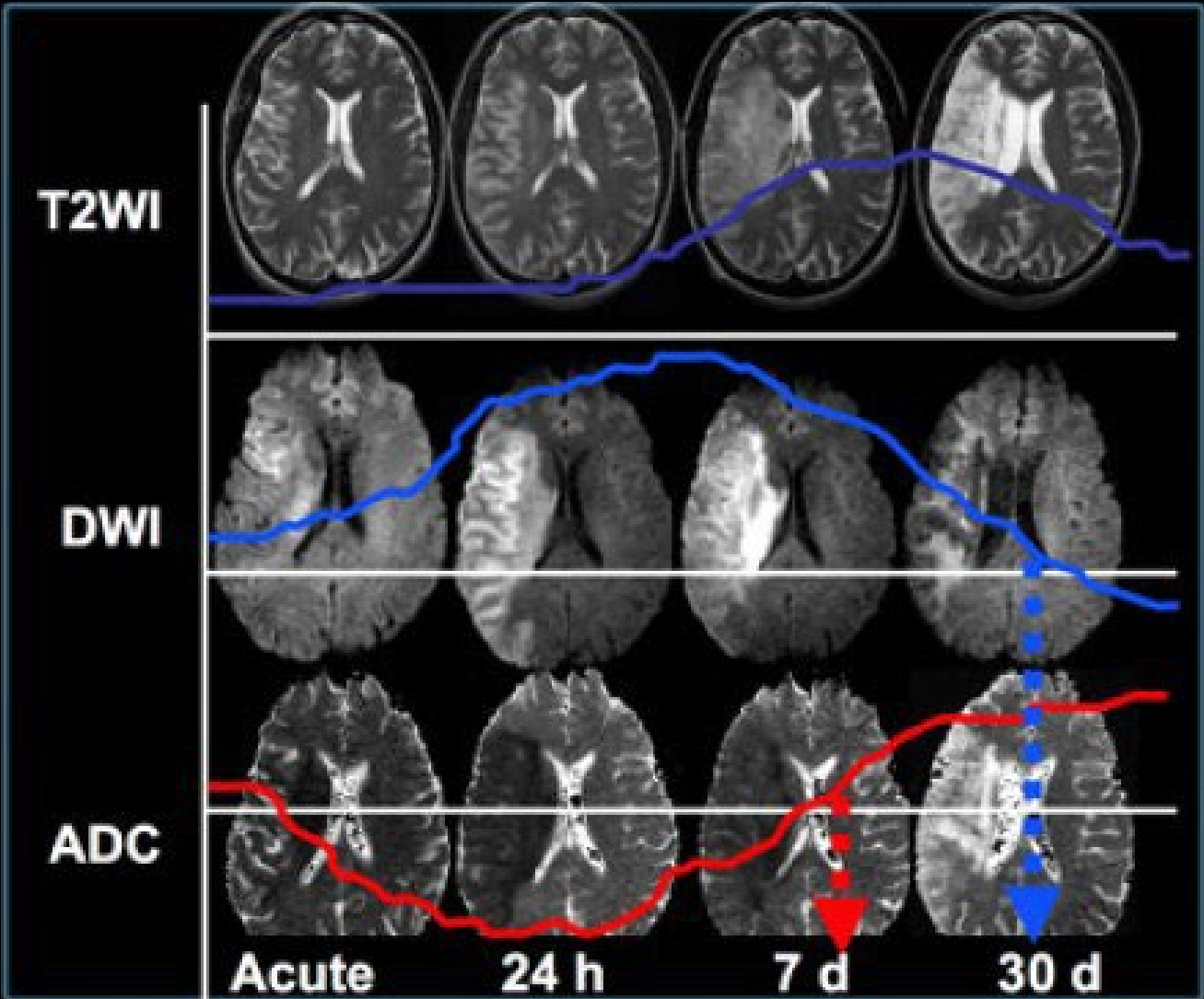
# What is diffusion and why do we care ?

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- Degree of diffusion restriction **affected by**
  - changes in the cellular density of tissue
  - amount of intracellular versus extracellular water
- Conditions such as :
  - ischemic infarcts, tumor **produce** highly restricted diffusion
  - cysts and edema **yield** low degrees of diffusion restriction

Diagnosis of pathological and histological information

# Diffusion Weighted MRI in clinic

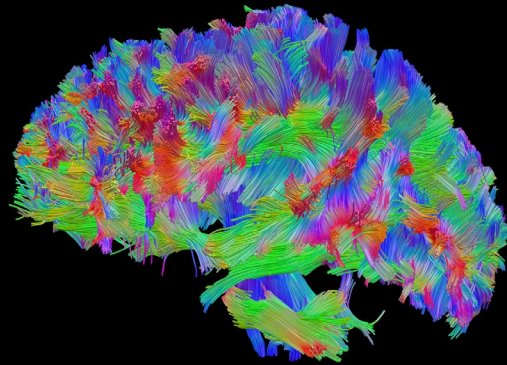


- Many pathologies cause restricted extracellular water diffusion
- Vascular Etiologies
  - *Ischemia-infarction*
  - *Hypoxic-ischemic Injury*
  - *Acute Hypertensive Encephalopathy*
  - *Venous Infarction*
- Infectious Etiologies
  - *Abscess*
  - *Empyema*
  - *Ventriculitis*
  - *Viral Encephalitis (Herpes, HIV, etc.)*
  - *Progressive Multifocal Leukoencephalopathy*
  - *Creutzfeldt-Jakob Disease*
  - *Toxoplasmosis*
- Neoplastic Etiologies
  - *Meningioma*
  - *Primary CNS Lymphoma*
  - *Glioblastoma*
- Demyelinating Etiologies
  - *Tumefactive Multiple Sclerosis*
  - *Neuromyelitis Optica*
  - *Acute Disseminated Encephalomyelitis*
- Metabolic/Toxic Etiologies
  - *Osmotic Demyelination Syndrome*
  - *Hypoglycemic Encephalopathy*
  - *Wernicke Encephalopathy*
  - *Carbon Monoxide Poisoning*
  - *Ethylene Glycol and Methanol Toxicity*
- Trauma
  - *Diffuse Axonal Injury*
- Miscellaneous Etiologies
  - *Epidermoid Cyst*
  - *Choroid Plexus Cyst*
  - *Status Epilepticus*
  - *Wallerian Degeneration*



- Anisotropy of water diffusion

Axonal pathways



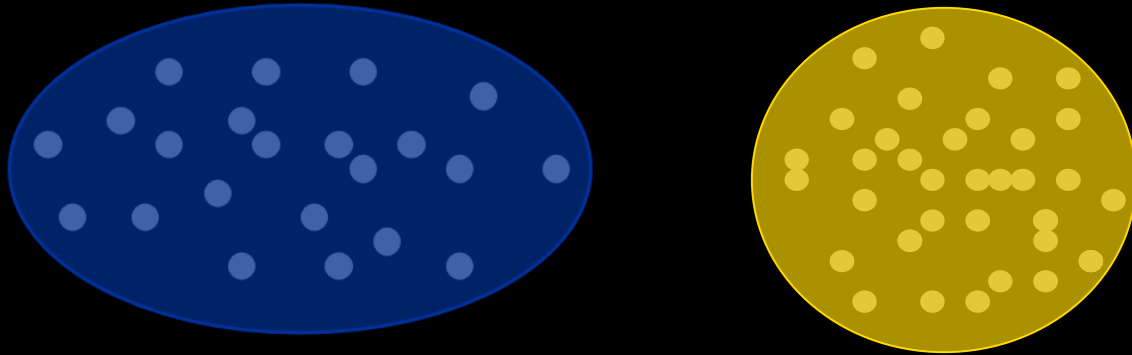
- in vivo connectivity (e.g., orientation, density)

- in vivo white matter integrity

(e.g.:, maturation/myelination, neurodegeneration)

- What is diffusion and why do we care ?
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- Full diffusion behavior in 3D space

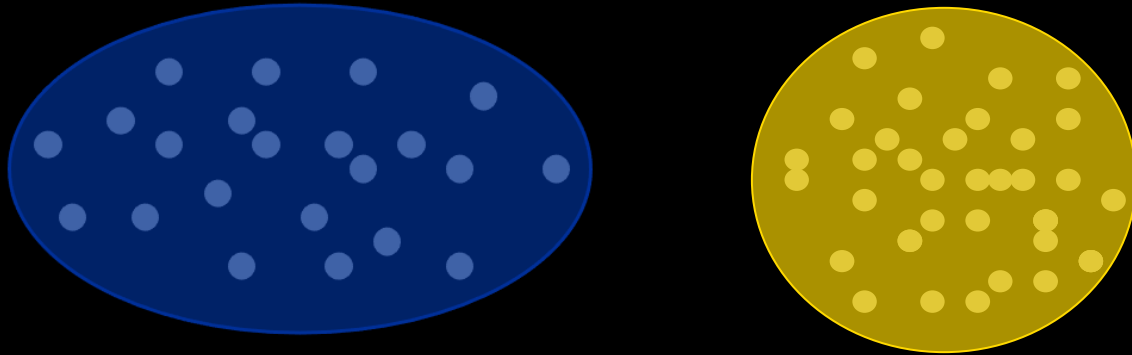


- described using ellipsoids (Tensor)

$$\underline{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

- Each element captures the variance of the diffusion process

- Full diffusion behavior in 3D space



- Diffusion Tensor

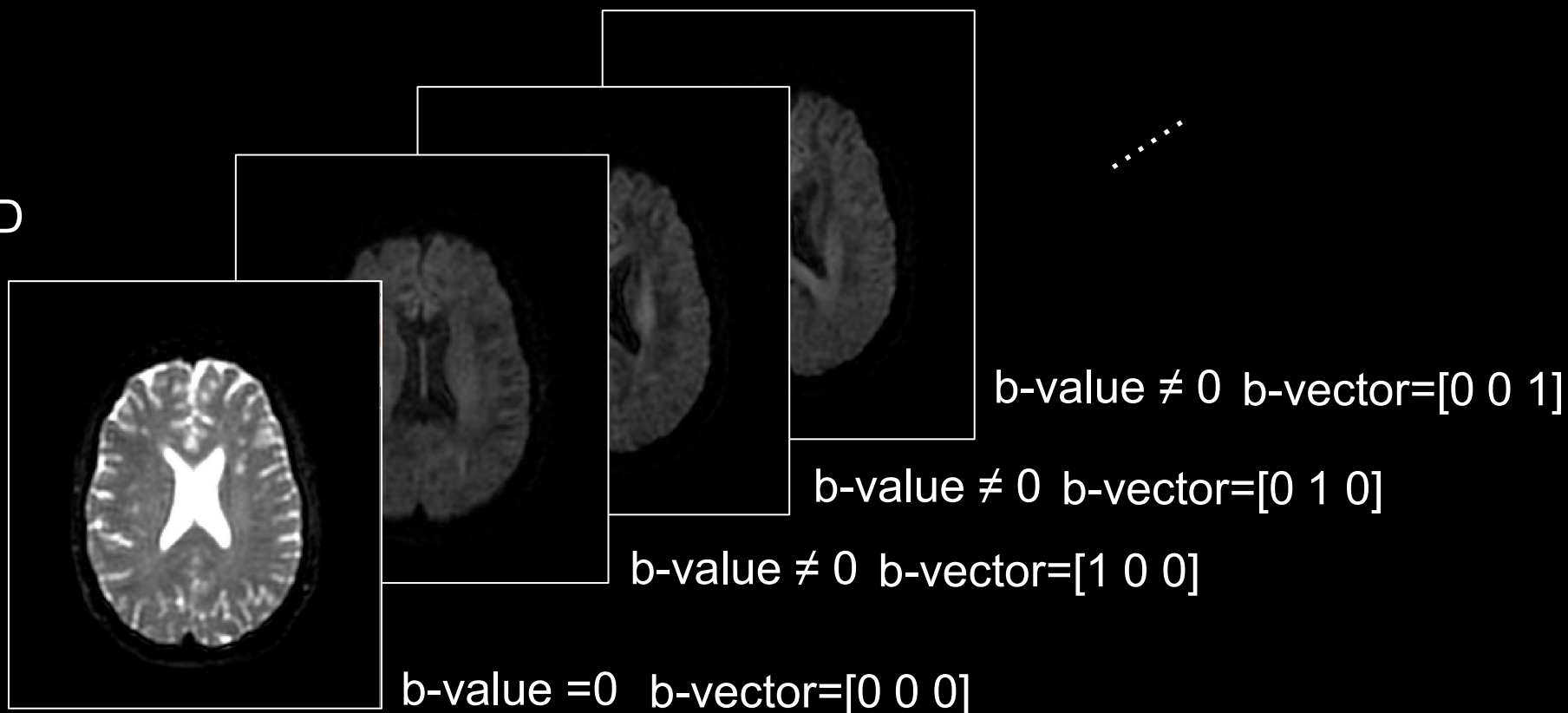
$$\underline{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

- Symmetric positive definite matrix
- 6 unknowns
- At least six measurements needed
- > 6 measurements will provide robustness to noise

b-value, b-vector

$$S_{u_i} = S_{ref} \exp(-bu_i^T \mathbb{D}u_i)$$

Dataset : 4D



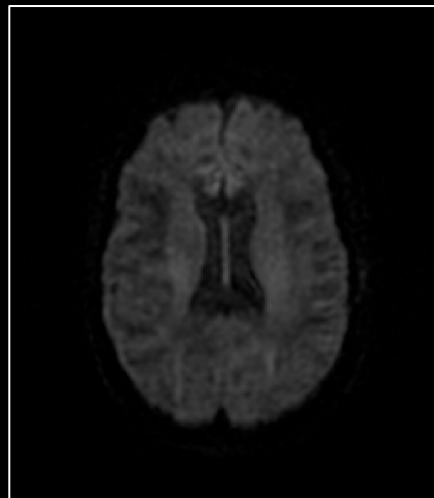
b-value, b-vector

$$S_{u_i} = S_{ref} \exp(-bu_i^T \mathbb{D}u_i)$$

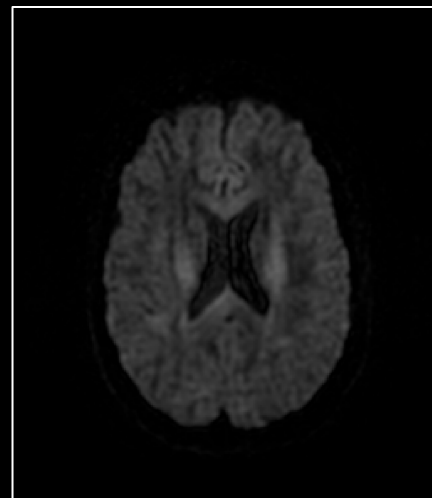
$S_{ref}$



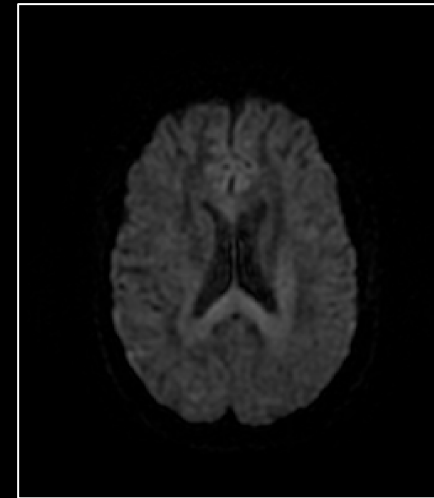
$S_{u_1}$



$S_{u_2}$



$S_{u_3}$



.....

b-vector =  
 $[u_{x1} \ u_{y1} \ u_{z1}]$

E.g.:  $[1 \ 0 \ 0]$

b-vector =  
 $[u_{x2} \ u_{y2} \ u_{z2}]$

E.g.:  $[0 \ 1 \ 0]$

b-vector =  
 $[u_{x3} \ u_{y3} \ u_{z3}]$

E.g.:  $[0 \ 0 \ 1]$

b-value, b-vector

$$S_{u_i} = S_{ref} \exp(-b u_i^T \mathbb{D} u_i)$$

$$-\frac{1}{b} \ln\left(\frac{S_{u_i}}{S_{ref}}\right) = \begin{bmatrix} u_{xi} & u_{yi} & u_{zi} \end{bmatrix} \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix} \begin{bmatrix} u_{xi} \\ u_{yi} \\ u_{zi} \end{bmatrix}$$

$$y_i = u_{xi}u_{xi}D_{xx} + u_{yi}u_{yi}D_{yy} + u_{zi}u_{zi}D_{zz} + 2u_{xi}u_{yi}D_{xy} + 2u_{yi}u_{zi}D_{yz} + 2u_{zi}u_{xi}D_{zx}$$

# Modeling diffusion

$$y_1 = u_{x1}u_{x1}D_{xx} + u_{y1}u_{y1}D_{yy} + u_{z1}u_{z1}D_{zz} + 2u_{x1}u_{y1}D_{xy} + 2u_{y1}u_{z1}D_{yz} + 2u_{z1}u_{x1}D_{zx}$$

$$y_2 = u_{x2}u_{x2}D_{xx} + u_{y2}u_{y2}D_{yy} + u_{z2}u_{z2}D_{zz} + 2u_{x2}u_{y2}D_{xy} + 2u_{y2}u_{z2}D_{yz} + 2u_{z2}u_{x2}D_{zx}$$

⋮

$$y_6 = u_{x6}u_{x6}D_{xx} + u_{y6}u_{y6}D_{yy} + u_{z6}u_{z6}D_{zz} + 2u_{x6}u_{y6}D_{xy} + 2u_{y6}u_{z6}D_{yz} + 2u_{z6}u_{x6}D_{zx}$$

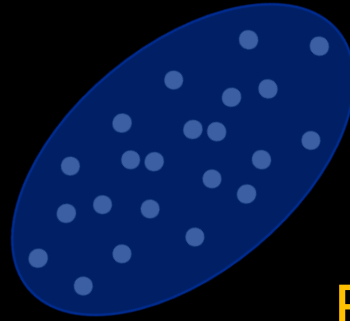
$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_6 \end{bmatrix}}_Y = \underbrace{\begin{bmatrix} u_{x1}u_{x1} & u_{y1}u_{y1} & \dots & 2u_{z1}u_{x1} \\ u_{x2}u_{x2} & u_{y2}u_{y2} & \dots & 2u_{z2}u_{x2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{x6}u_{x6} & u_{y6}u_{y6} & \dots & 2u_{z6}u_{x6} \end{bmatrix}}_A \underbrace{\begin{bmatrix} D_{xx} \\ D_{yy} \\ \vdots \\ D_{zx} \end{bmatrix}}_X \quad X = A^{-1}Y$$

Many software programs have built-in tools to perform a DTI fitting



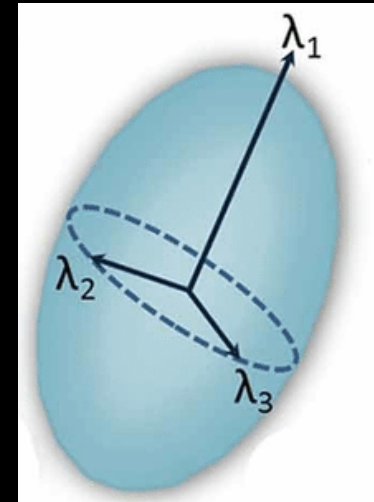
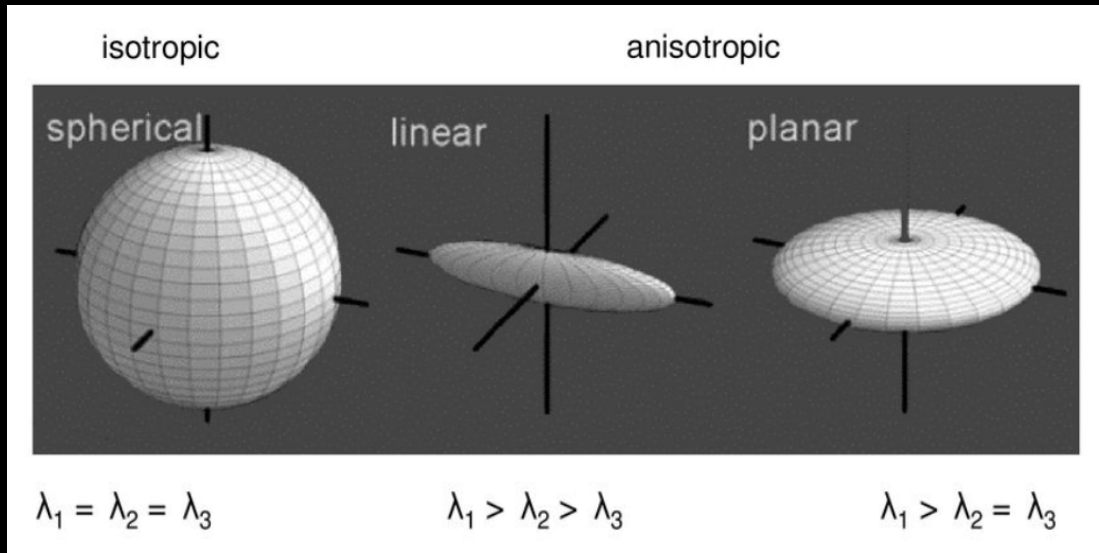
# Anisotropy : Major axis of the ellipsoid

$$\underline{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$



$$\mathbb{D} = E \Lambda E^T = E^T \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} E$$

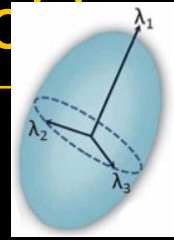
Eigen Decomposition



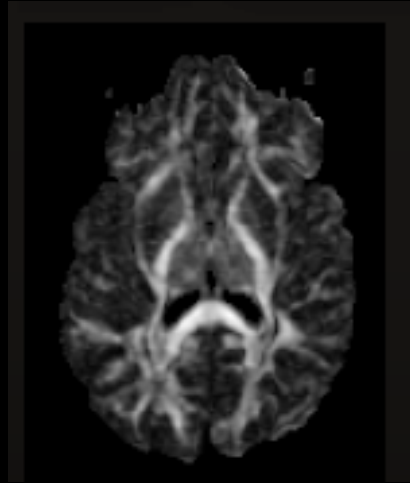
Eigenvectors

$$E = \begin{bmatrix} e_{1x} & e_{2y} & e_{3z} \\ e_{1x} & e_{2y} & e_{3z} \\ e_{1x} & e_{2y} & e_{3z} \end{bmatrix}$$

# Anisotropy : Major axis of the ellipsoid



For every voxel



DWIs

$$\underline{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

Solve for D

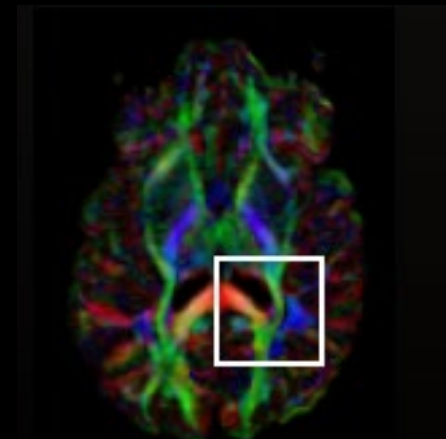
$$E^T \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} E$$

Eigen Decomposition

Eigenvectors

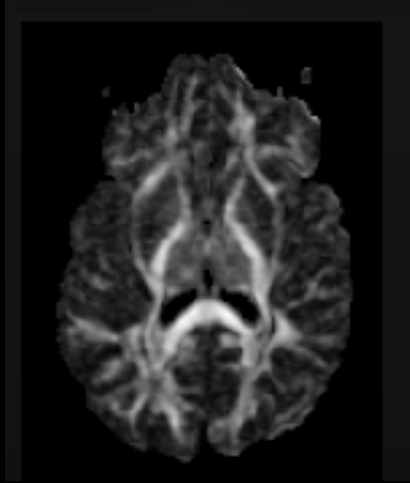
$$E = \begin{bmatrix} e_{1x} & e_{2y} & e_{3z} \\ e_{1x} & e_{2y} & e_{3z} \\ e_{1x} & e_{2y} & e_{3z} \end{bmatrix}$$

Assign Color-code to the Eigen vectors



# Anisotropy : Major axis of the ellipsoid

For every voxel



DWIs

$$\underline{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

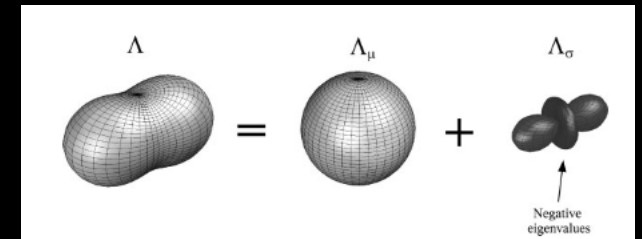
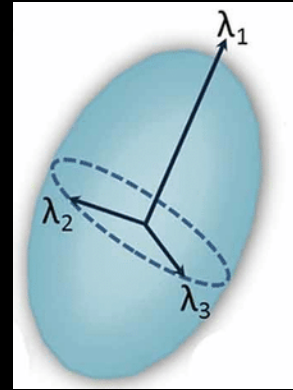


Solve for D

$$E^T \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} E$$



Eigen  
Decomposition



$$FA = \sqrt{\frac{3}{2}} \frac{\sqrt{\Lambda_\sigma : \Lambda_\sigma}}{\sqrt{\Lambda : \Lambda}}$$

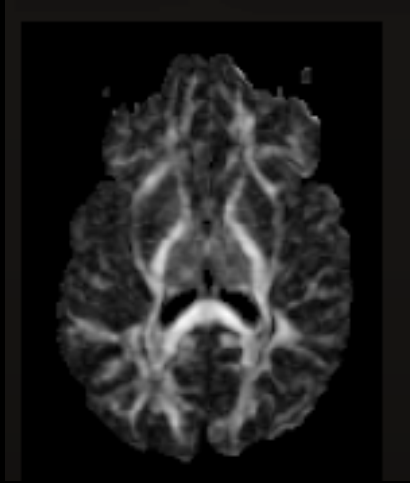
**Fractional Anisotropy Map**

$$FA = \sqrt{\frac{1}{2}} \cdot \frac{\sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2}}{\sqrt{(\lambda_1)^2 + (\lambda_2)^2 + (\lambda_3)^2}}$$

Ranges from values : 0 ->1

1 : high anisotropy, 0: low anisotropy

For every voxel



DWIs

$$\underline{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix}$$

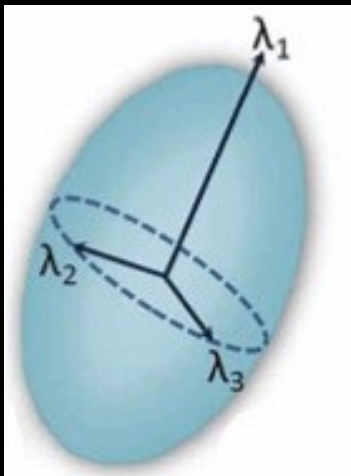
$$E^T \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} E$$



Solve for D



Eigen  
Decomposition



Axial Diffusivity:  $\lambda_1$

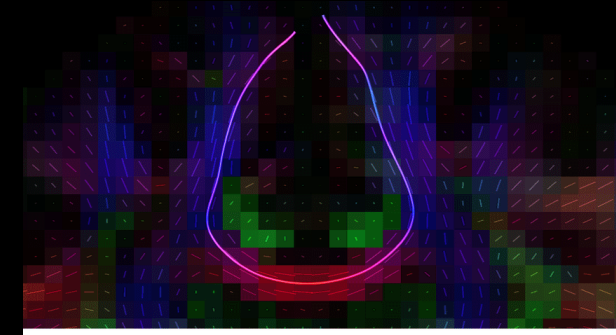
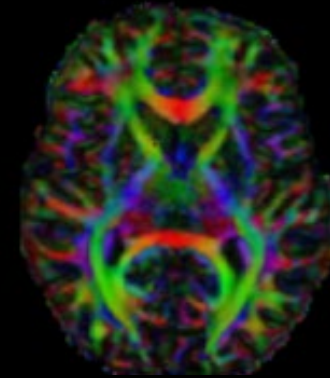
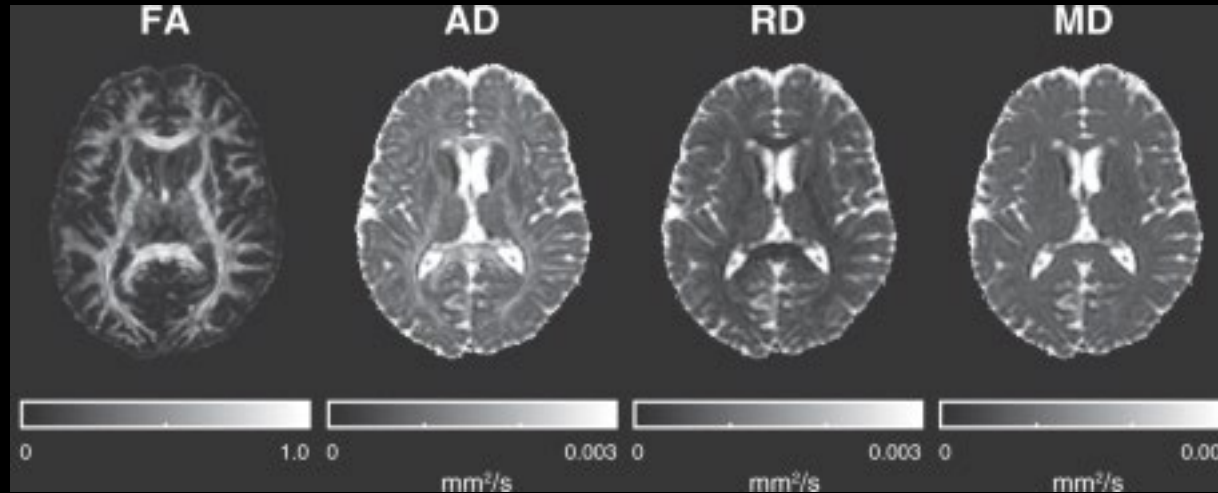
Radial Diffusivity:  $\frac{\lambda_2 + \lambda_3}{2}$

Mean Diffusivity:  $\frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$

Range: 0 - .003 mm<sup>2</sup>/sec

# Diffusion Tensor Imaging

- With 6 measurements, diffusion in 3D space is characterized



Enables studies of brain disorders

Is there change of white matter integrity ?

Is there change in connectivity patterns?

## Myth about FA

**✗ Lower FA means loss of WM integrity  
(and vice versa)**

**✓ Lower FA can result from increased WM  
integrity also (and vice versa)**

## Myth about DTI

✗ DTI provides measures of WM only

✓ All DTI maps are valid in both WM and GM

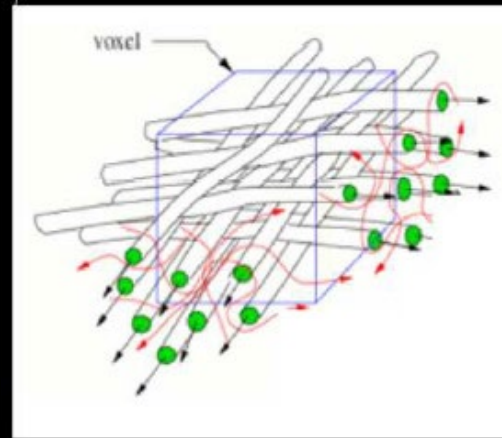
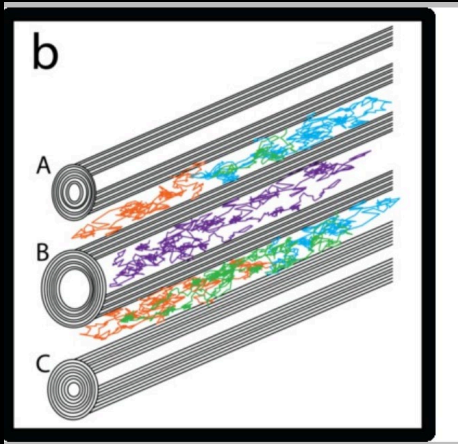
✓ Changes in FA are more frequently “tried” to be interpreted in WM

✓ DTI can detect changes in GM also

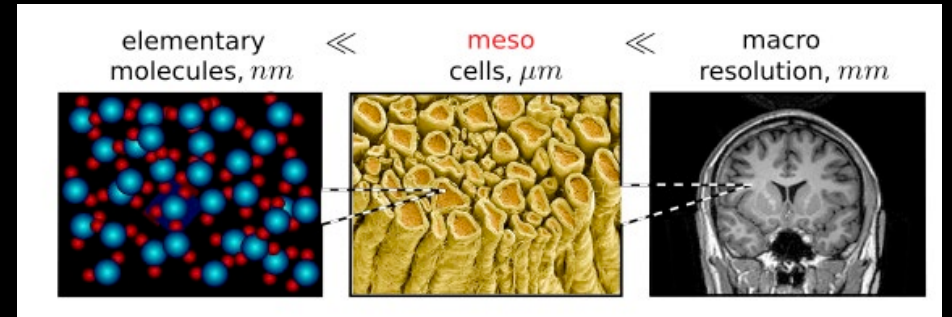
- What is diffusion and why do we care ?
- How to model diffusion signal?
- What are the pitfalls of modeling ?
- Fiber Tracking using diffusion MRI
- How to pre-process diffusion data?
- How to extract biologically meaningful diffusion measures ?



- Diffusion Tensor is a over-simplistic model for brain studies
  - With unparalleled sensitivity

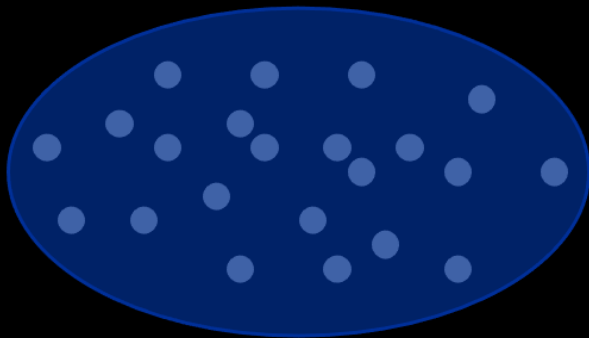


Most MRI voxels are  $> 8\text{mL}$



Axons are orders of magnitude smaller structures

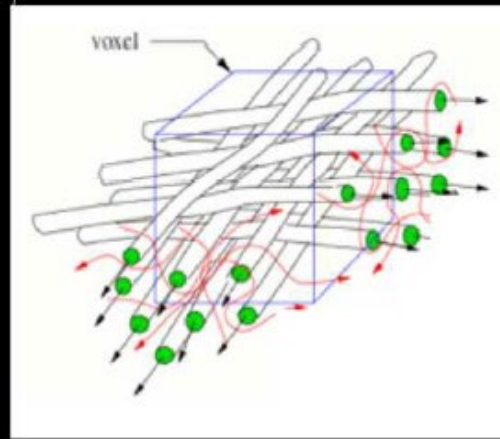
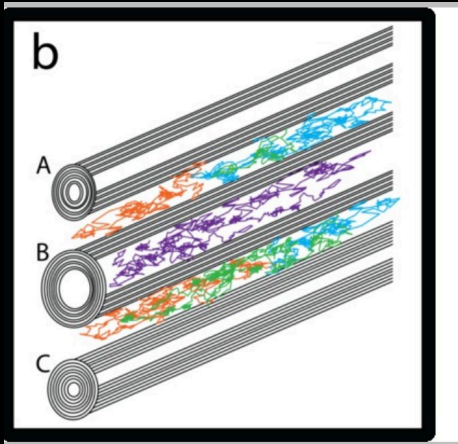
??



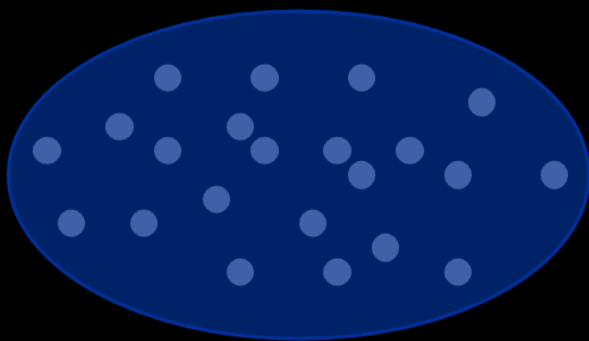
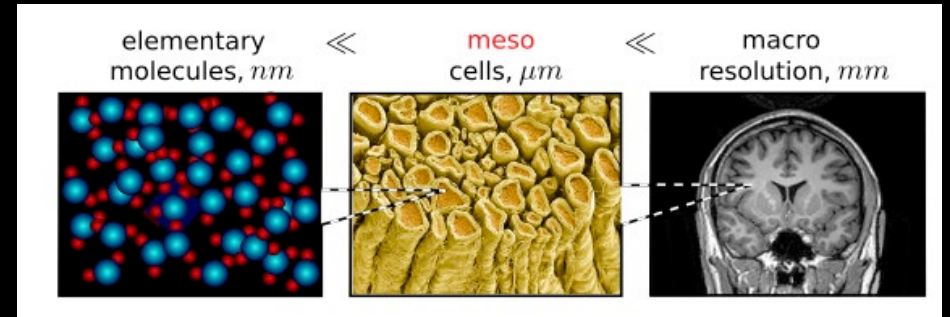
Anisotropic Diffusion

Key word: “change”

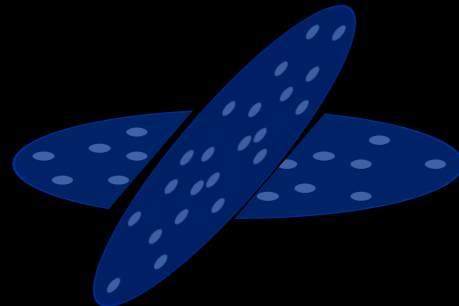
- Diffusion Tensor is a over-simplistic model for brain studies
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Most MRI voxels are  $> 8\text{mL}$



Anisotropic Diffusion



Multiple ellipsoids

Hard to solve mathematically

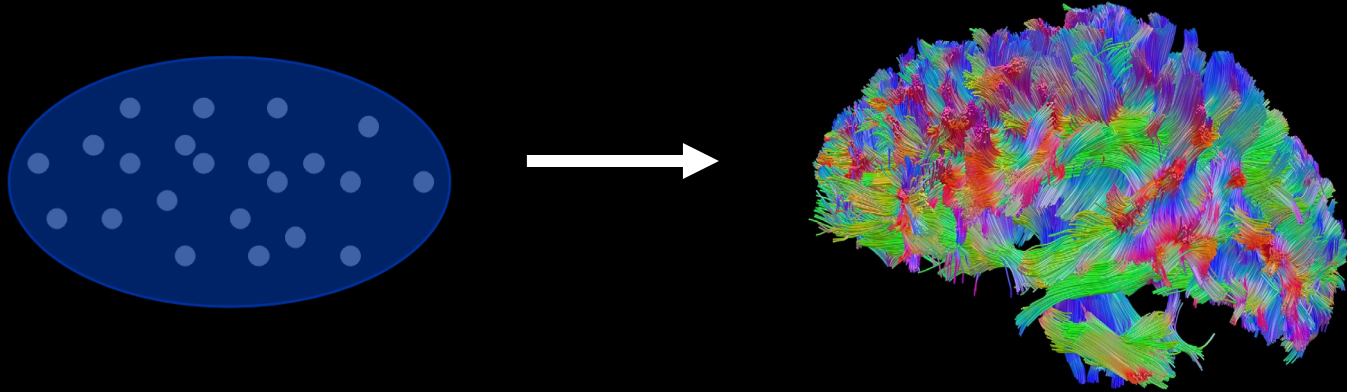
## Fact about DTI

- ✓ DTI can be used to study brain changes
- ✓ DTI is not a good model for anisotropy detection in most brain voxels
- ✓ Because of fiber orientation heterogeneity

- What is diffusion and why do we care ?
- How to model diffusion ?
- What are the pitfalls of modeling ?
- **Fiber Tracking using diffusion MRI**
- How to pre-process diffusion data?
- How to extract biologically meaningful diffusion measures ?

# Fiber Tracking Using Diffusion data

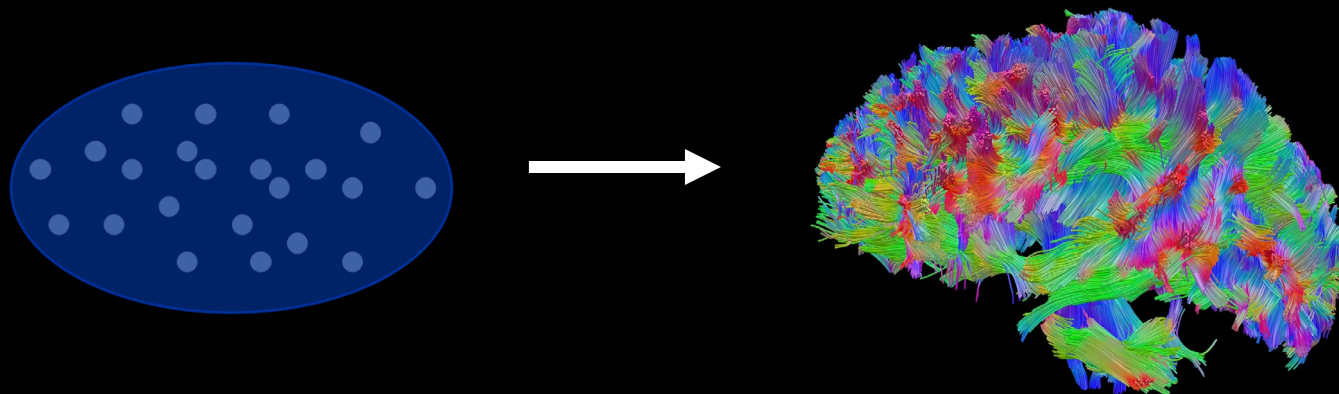
- If the ellipsoid model is not good, how can you do fiber tracking ??



- Modern day fiber tracking is not performed using ellipsoids
- Makes use of a method called **spherical deconvolution**

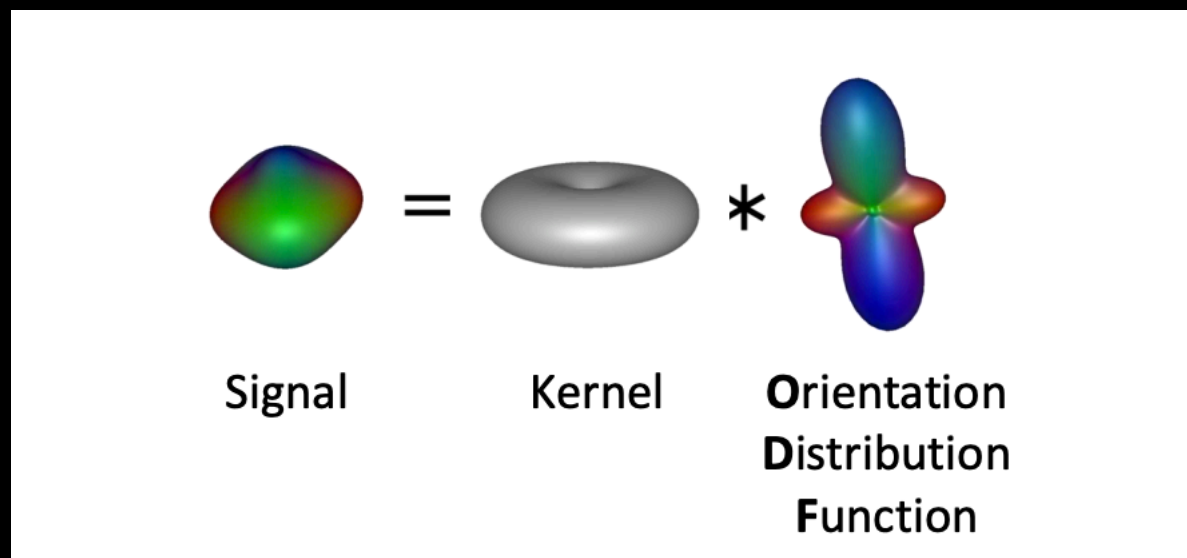
# Fiber Tracking Using SD

- If the ellipsoid model is not good, how can you do fiber tracking ??



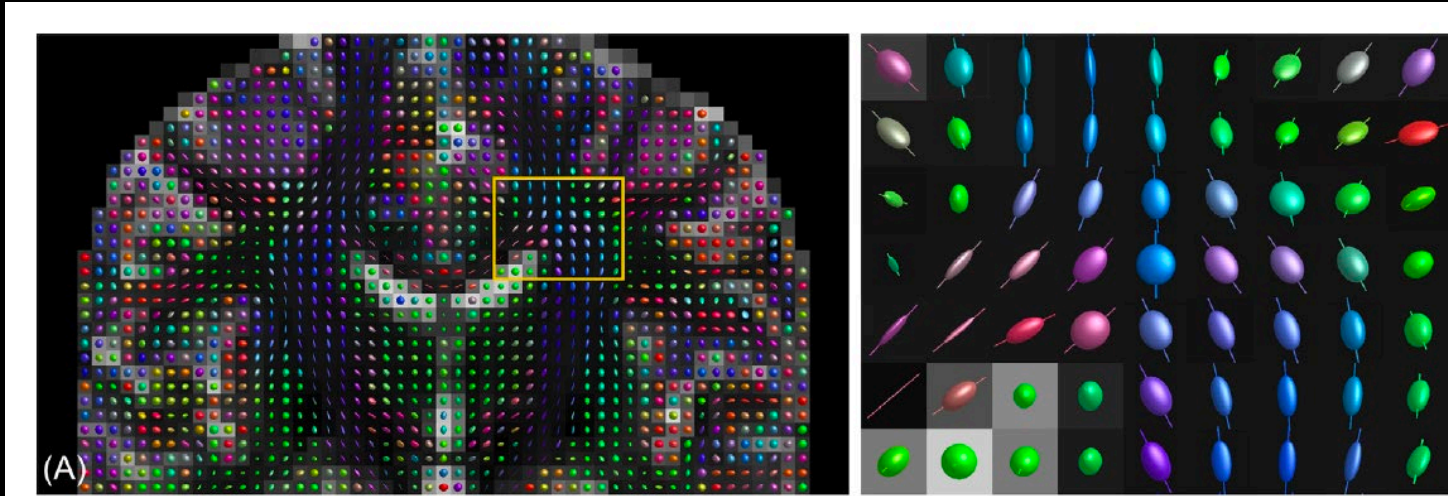
- Modern day fiber tracking is not performed using ellipsoids
- Makes use of a method called **spherical deconvolution**
- Only pertains to finding the fiber orientations
- Do not provide any information about the diffusivity
- Makes certain assumptions

## ➤ SD Model

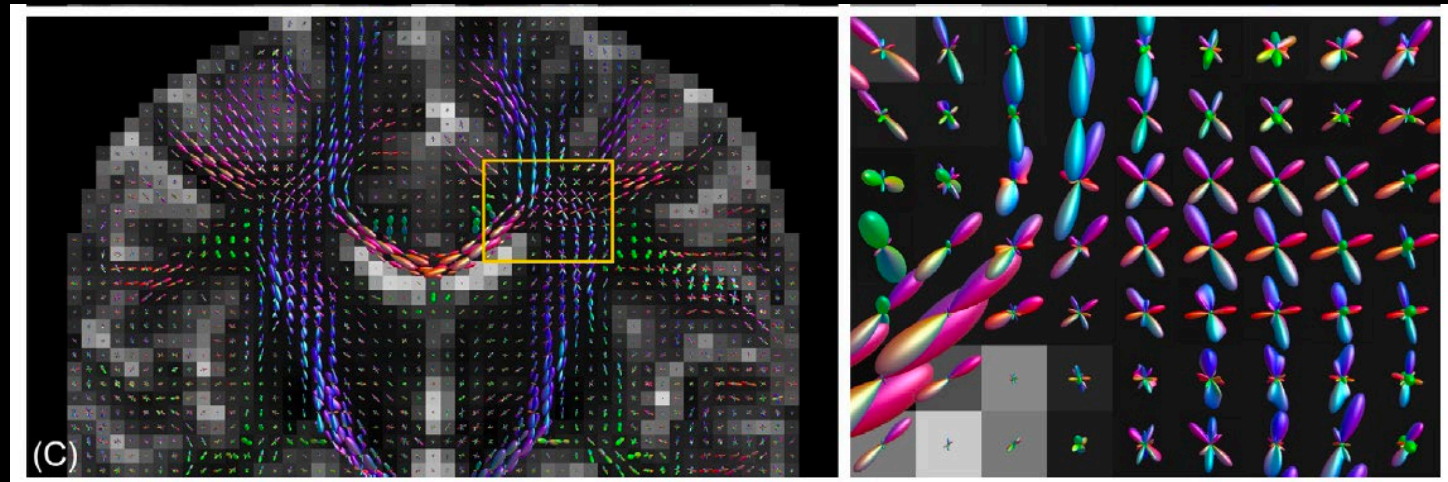


- Measured diffusion signal is a convolution
  - Of a known fiber response kernel
  - With a fiber orientation distribution function (fODF)
- Goal: Given the kernel, find the fiber ODF

# Spherical Deconvolution



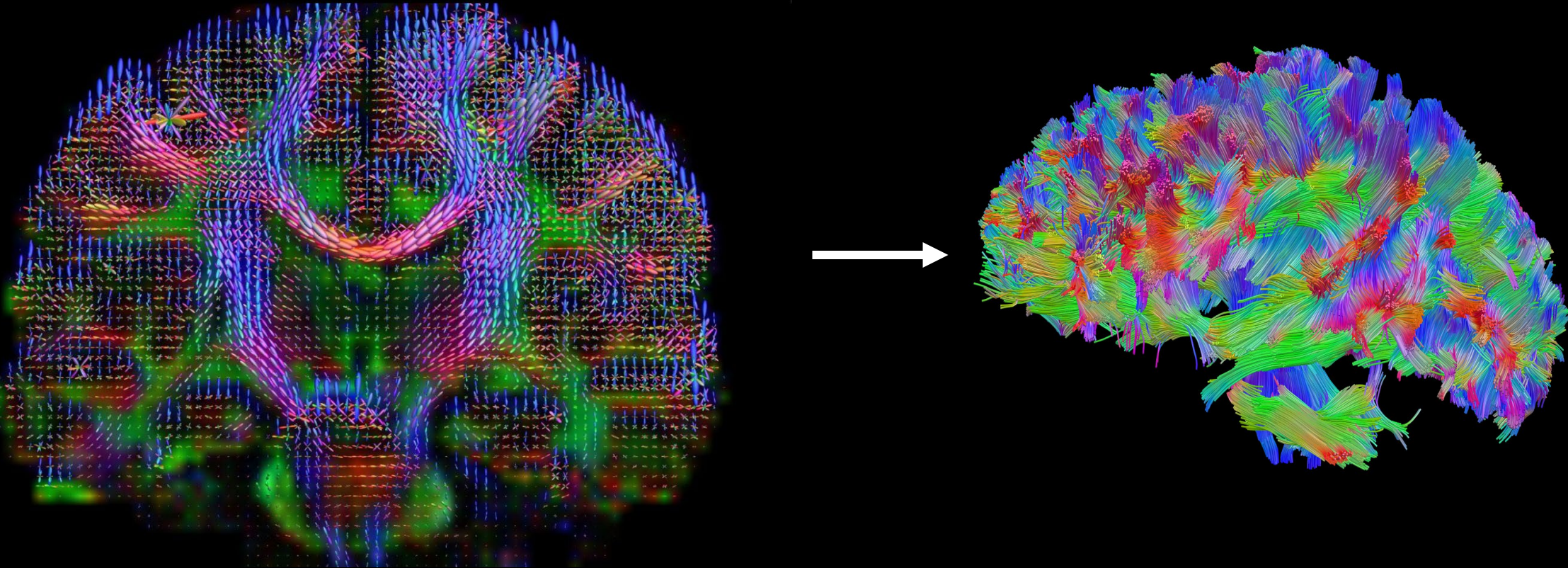
➤ Tensor fitting



➤ SD



# Fiber Tracking Using Diffusion data



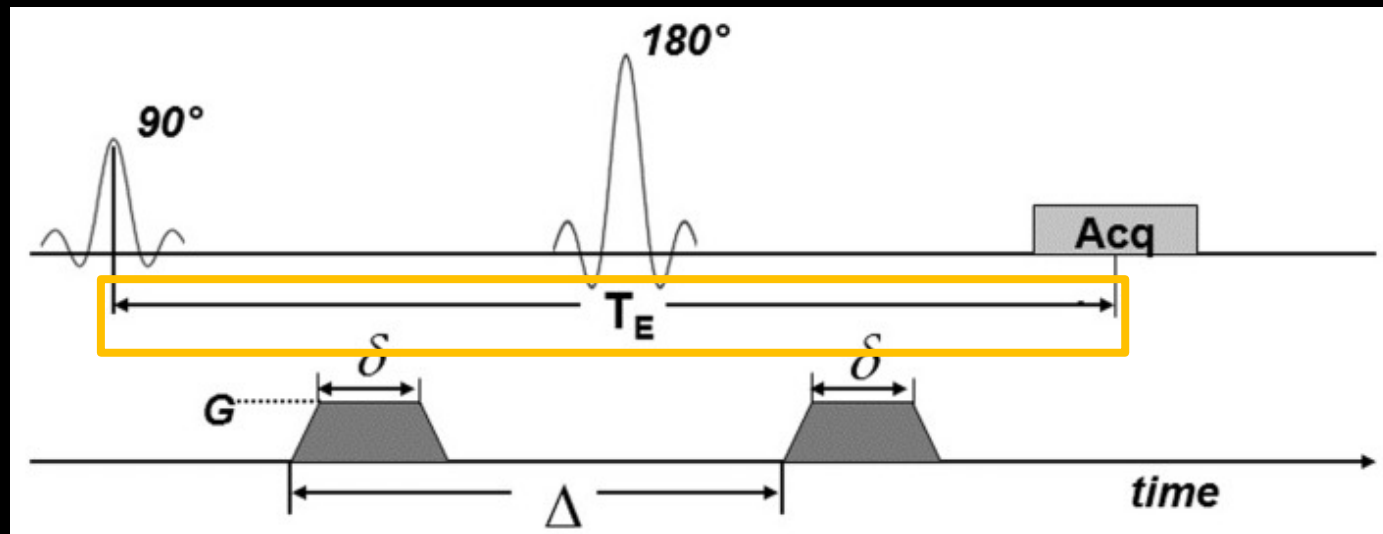
- Implemented in many software packages
  - MrTrix, DIPY

- What is diffusion and why do we care ?
- How to model diffusion signal?
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- DW images are affected by **noise** and by artifacts such as **Gibbs ringing** and **distortions**.

# Noise in diffusion MRI

- DW images are affected by **noise** and by artifacts such as **Gibbs ringing** and **distortions**.
- Noise: diffusion MRI has very low SNR



- TE definition

# Noise in diffusion MRI

---

- DW images are affected by **noise** and by artifacts such as **Gibbs ringing** and **distortions**.
- Noise: diffusion MRI has very low SNR
- Acquisition parameters that affect the SNR:
  - b-value (which affects the TE)
  - Spatial resolution (which affects the TE)
  - The scanner itself ( $G_{\max}$ ,  $Slew_{\max}$ )

# Denoising

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- Many correction methods are available:
  - Local-PCA, Marchenko Pastur PCA
  - Patch2Self
  - Both implemented in DIPY

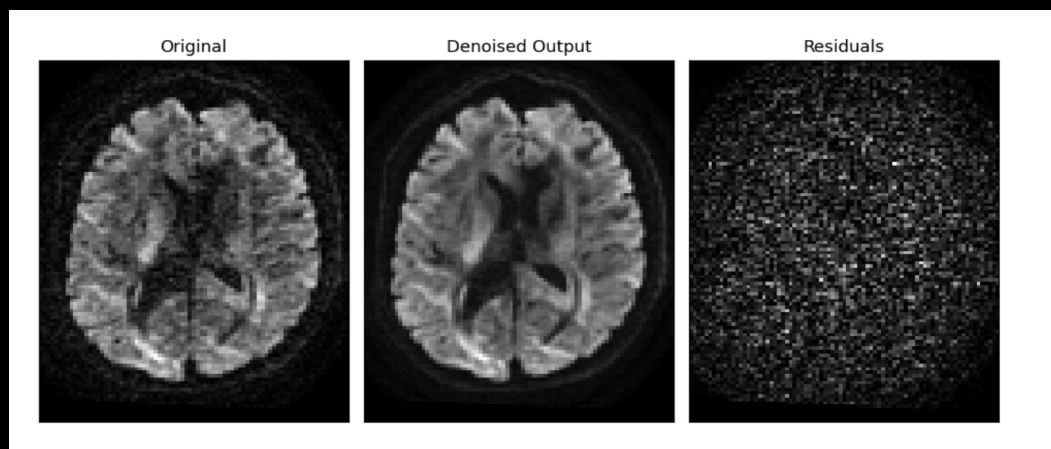
```
denoised_arr = mppca(data, patch_radius=2)
```

```
denoised_arr = patch2self(data, bvals)
```

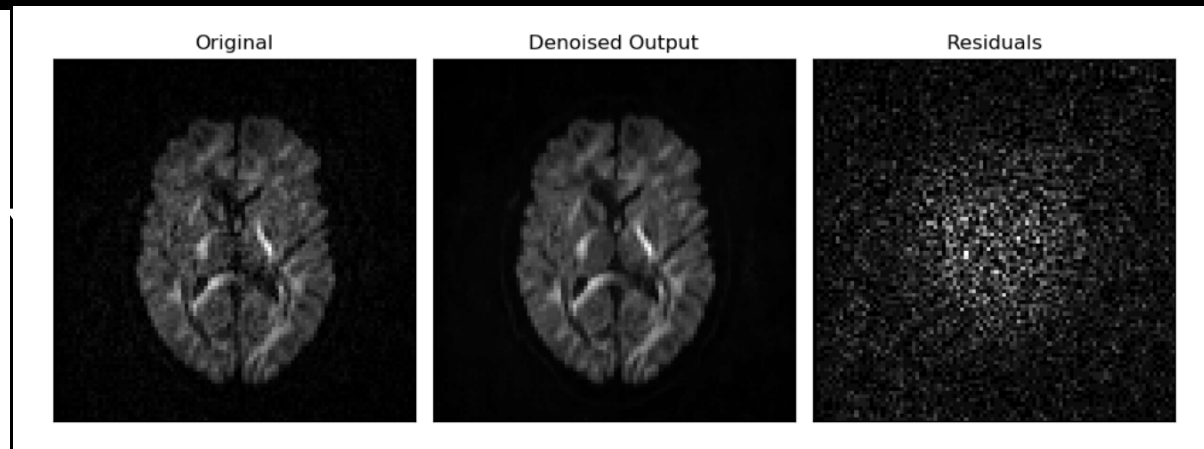
- NORDIC, qModeL, AIReconDL

# Denoising

- Many correction methods are available:
  - Local-PCA, Marchenko Pastur PCA
  - Patch2Self
  - Both implemented in DIPY



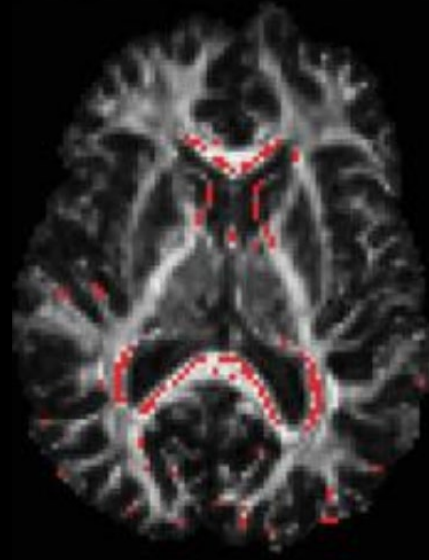
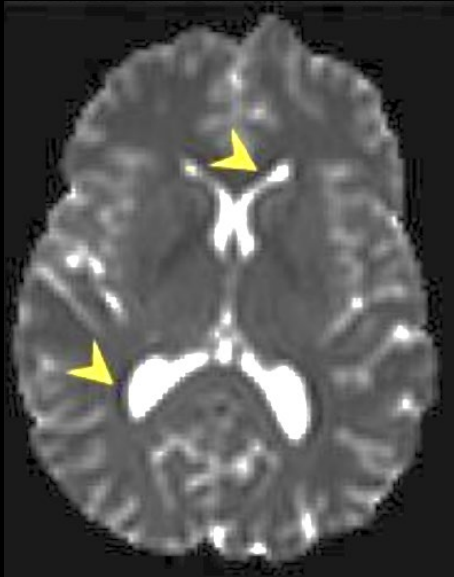
```
denoised_arr = mppca(data, patch_radius=2)
```



```
denoised_arr = patch2self(data, bvals)
```

# Gibbs Ringing

- DW images are affected by **noise** and by artifacts such as **Gibbs ringing** and **distortions**.
- Gibbs Ringing



- Caused by truncation of high frequency Fourier coefficients

- Usually most prominent on the b0 images
- Leads to over-estimation of FA ( $>1$ )



# Gibbs Ringing

- Correction needed to remove outliers
  - Sub-voxel shift method
    - Implemented in DIPY

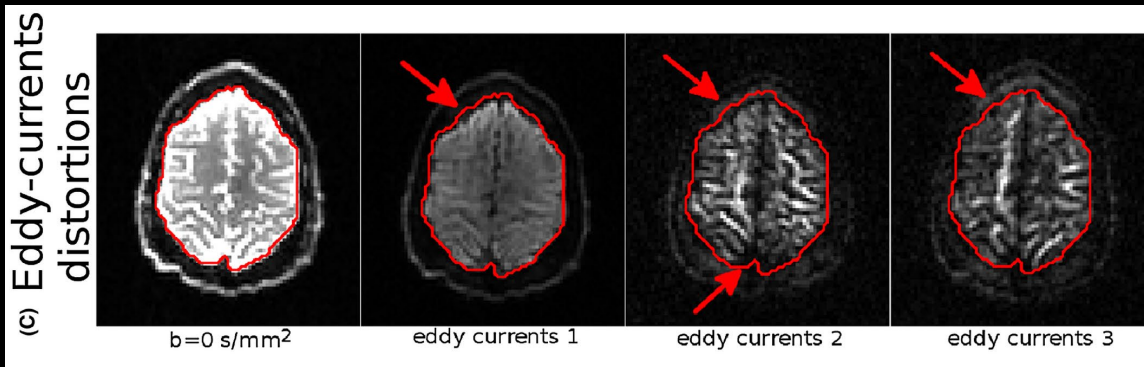
To run the Gibbs unringing on the data it suffices to execute the `dipy_gibbs_ringing` command, e.g.:

```
dipy_gibbs_ringing data/tissue_data/t1_brain_denoised.nii.gz --num_threads 4 --out_dir "gibbs_ringing_c"
```

- Advanced version for partial Fourier acquired data (RPG)
  - Implemented in DESIGNER <https://github.com/NYU-DiffusionMRI/DESIGNER>  
`designer dwi1.nii.gz out_dir -rpg -pf 6/8 -dim 2`

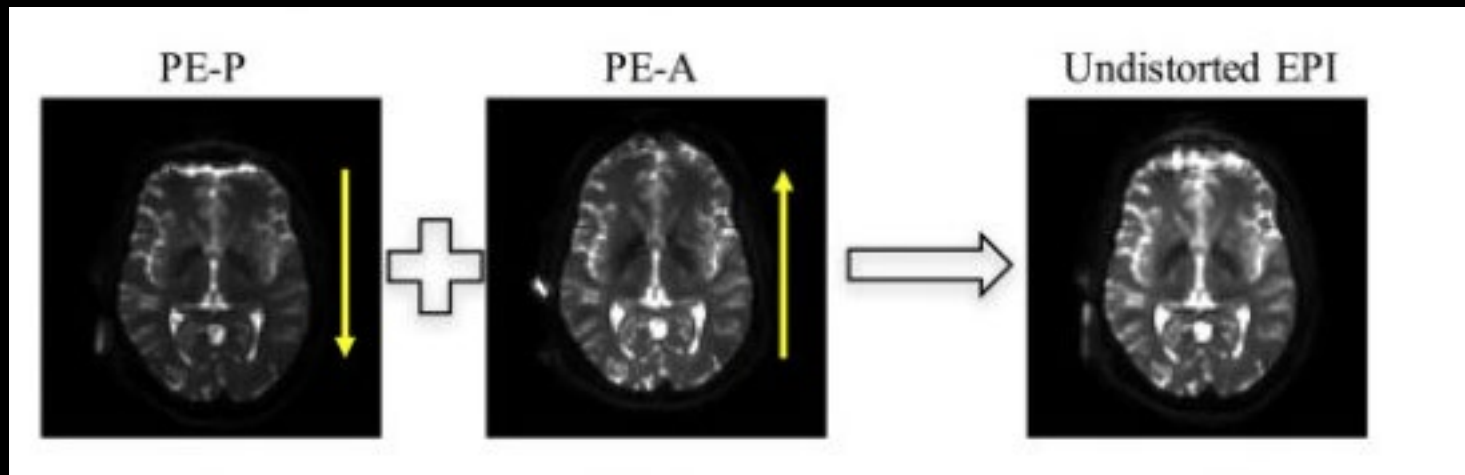
# Distortion Correction

- DW images are affected by **noise** and by artifacts such as **Gibbs ringing** and **distortions**
- Two sources:
  - B0 field inhomogeneity
  - Eddy current



# Distortion Correction

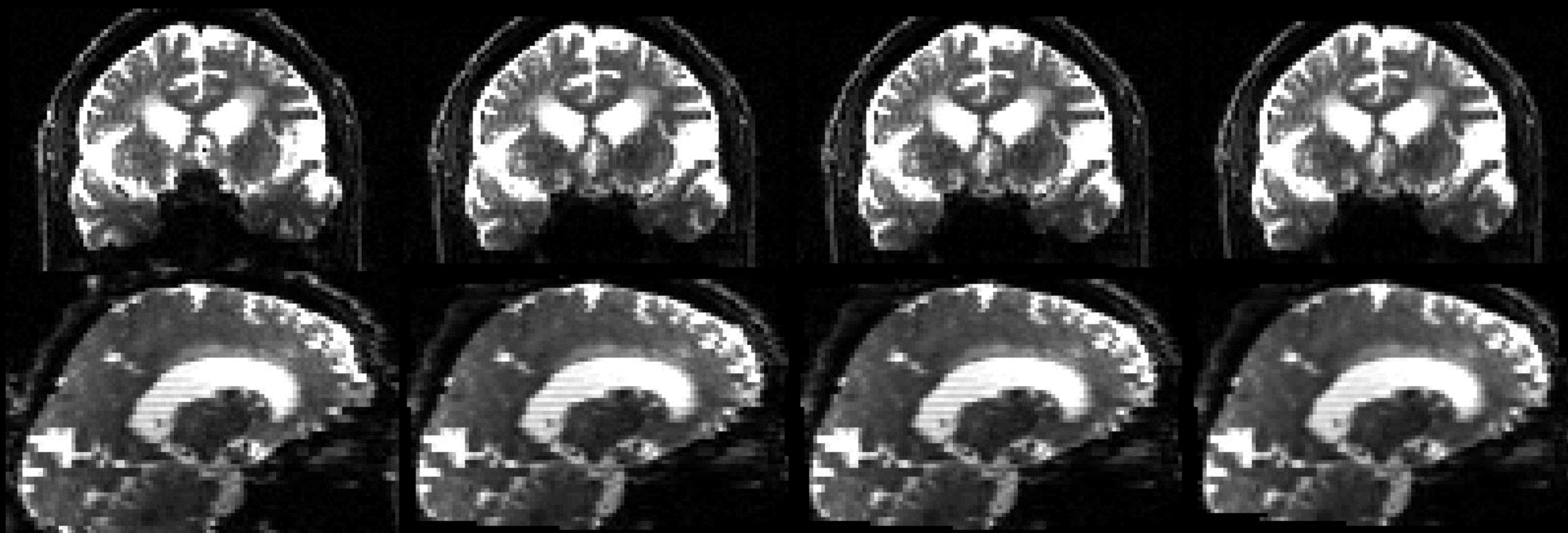
- Susceptibility-induced : top-up
- Two scans (b0s) acquired with opposite k-space read-out polarity are needed



```
topup --imain=all_my_b0_images.nii --datain=acquisition_parameters.txt --config=b02b0.cnf --out=my_output
```

# Distortion Correction

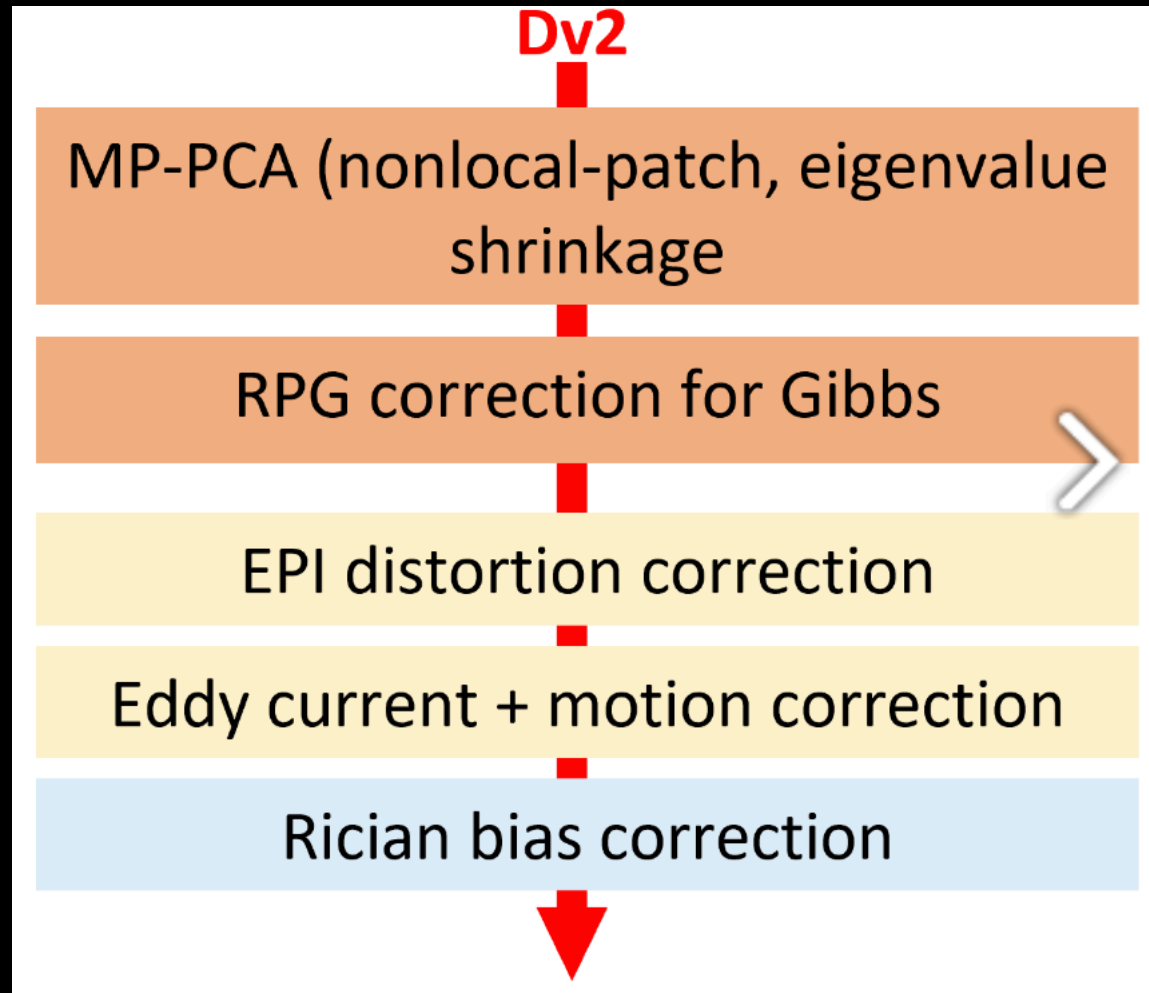
- Eddy-current induced : eddy
  - Also corrects for head motion, and signal dropout



# Bias Correction

---

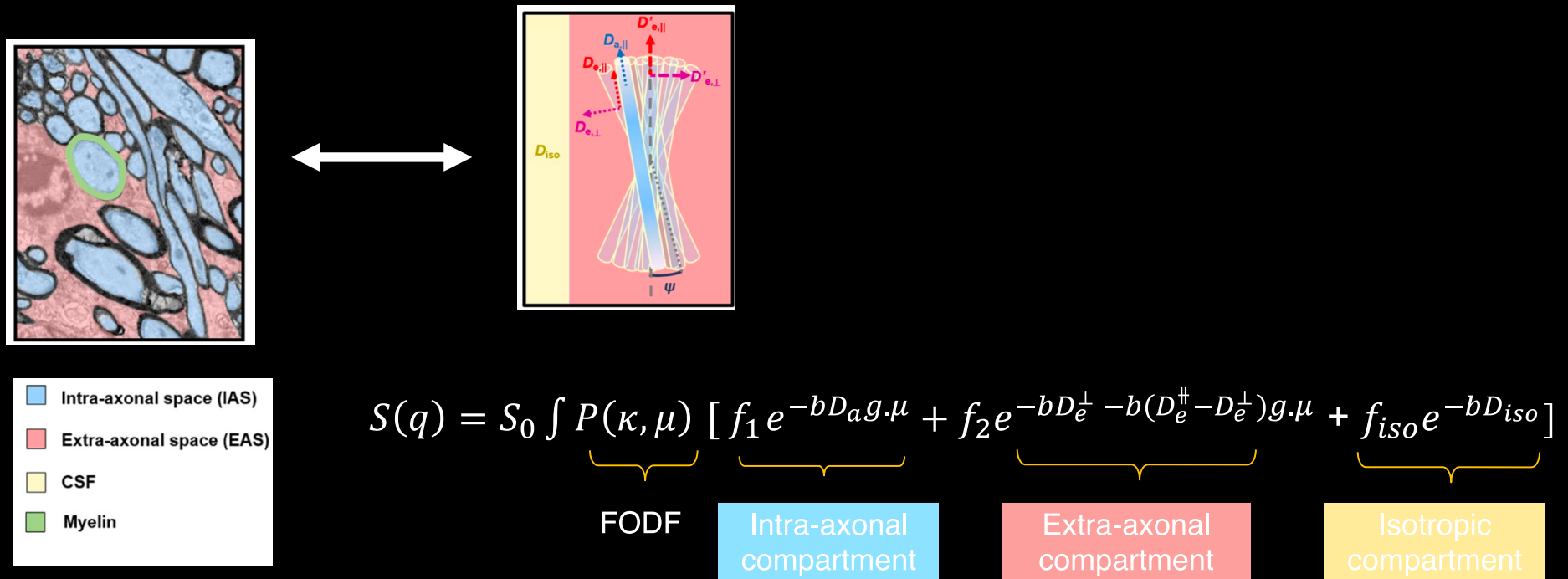
- Magnitude MRI images have a Rician noise distribution
  - Creates a bias in parameter estimates
  - Major concern at low SNR acquisitions
    - For  $b \sim 1000$ , not a big issue
    - For  $b > 1000$ , needs bias correction
  - Not many toolboxes have this implemented
  - Implemented in DESIGNER <https://github.com/NYU-DiffusionMRI/DESIGNER>  
`designer dwi1.nii.gz out_dir -denoise -rician`



- What is diffusion and why do we care ?
- How to model diffusion ?
- What are the pitfalls of modeling ?
- How to measure diffusion data?
- How to pre-process diffusion data?
- How to extract biologically meaningful diffusion measures ?

# Biologically meaningful diffusion measures

- Involve biophysical modeling
- Split a voxel into multiple compartments
- Measure diffusion properties associated with each compartment





# Biologically meaningful diffusion measures

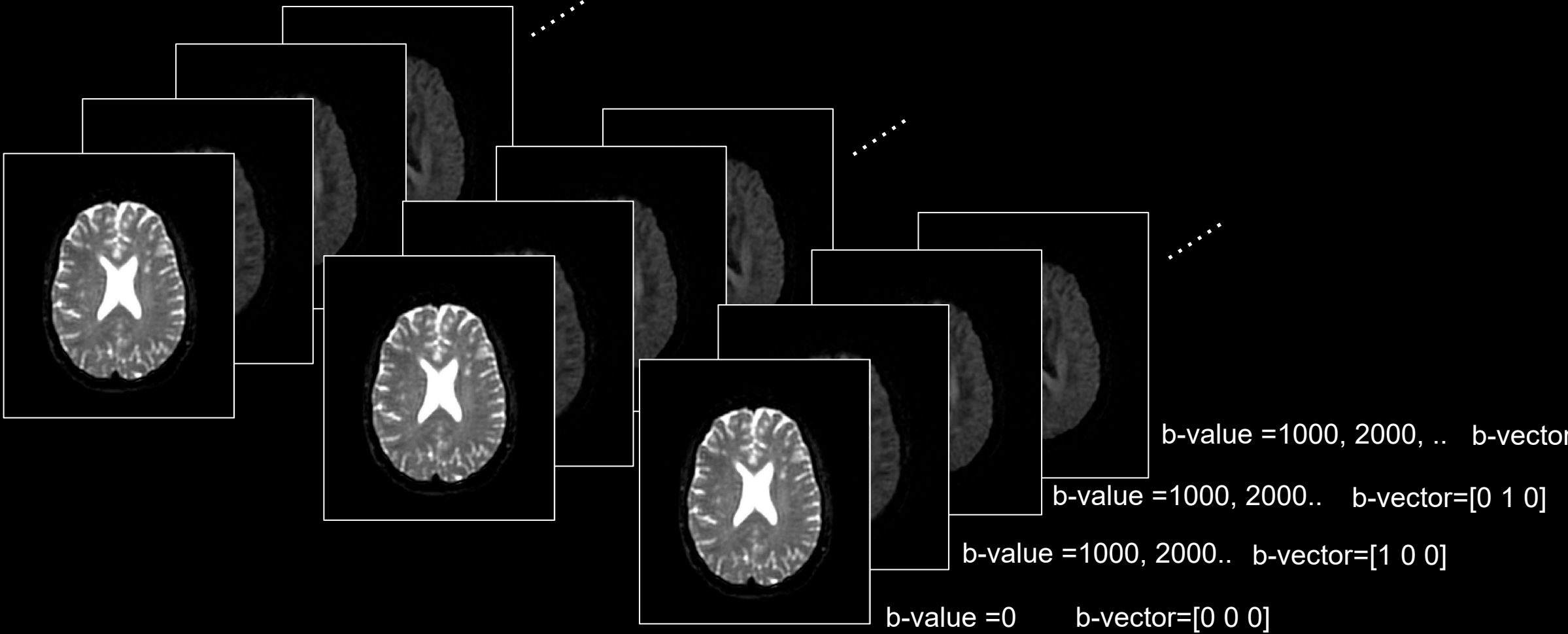
$$S(q) = S_0 \int P(\kappa, \mu) \left[ f_1 e^{-bD_a g \cdot \mu} + f_2 e^{-bD_e^\perp - b(D_e^\parallel - D_e^\perp) g \cdot \mu} + f_{iso} e^{-bD_{iso}} \right]$$

FODF

Intra-axonal  
compartmentExtra-axonal  
compartmentIsotropic  
compartment

- Large number of unknowns
- Requires multi-shell sampling
- Requires special MRI scanners
- HCP study was a big leap

# ➤ Multi-shell acquisition



# Biologically meaningful diffusion measures

$$S(q) = S_0 \int P(\kappa, \mu) \left[ f_1 e^{-bD_a g \cdot \mu} + f_2 e^{-bD_e^\perp - b(D_e^\parallel - D_e^\perp)g \cdot \mu} + f_{iso} e^{-bD_{iso}} \right]$$

FODF
Intra-axonal compartment
Extra-axonal compartment
Isotropic compartment

- Large number of unknowns
- Requires multi-shell sampling
- Requires special MRI scanners
- New dedicated scanner are now available for such studies
- MAGUS scanner installed at UIOWA in Dec 2022 !!

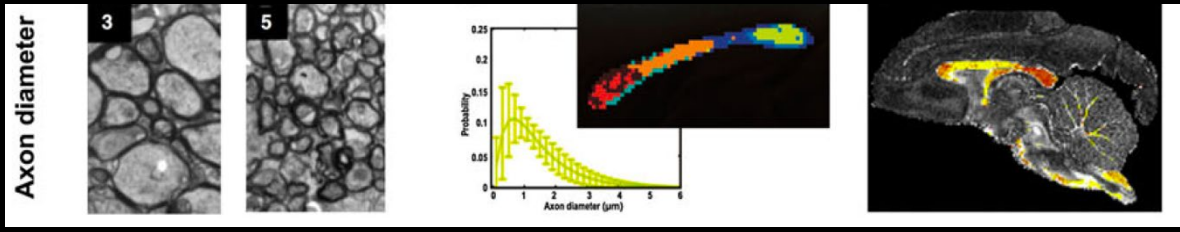
# Biologically meaningful diffusion measures

- Large number of biomarkers for neurodegeneration!

$$S(q) = S_0 \int \underbrace{P(\kappa, \mu)}_{\text{FODF}} \left[ \underbrace{f_1 e^{-bD_a g \cdot \mu}}_{\text{Intra-axonal compartment}} + \underbrace{f_2 e^{-bD_e^\perp - b(D_e^\parallel - D_e^\perp)g \cdot \mu}}_{\text{Extra-axonal compartment}} + \underbrace{f_{iso} e^{-bD_{iso}}}_{\text{Isotropic compartment}} \right]$$

Axonal loss
Axonal Beading
Demyelination
Neuroinflammation

- Axon diameter mapping



- Diffusion measures are highly sensitive
- DTI is a useful tool that exploits the high sensitivity
- DTI studies do not facilitate biological interpretation
- Biophysical models do facilitate biological interpretation
- Always collect multi-shell data when planning diffusion studies
- Requires dedicated scanner
- Hawkeyes can be proud to own the rarest scanner yet!
- Many tools are already available for testing on argon!
- Please plan to make use of it !!